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Simple Strategy for Powered Swingby

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The use of thrust during a close encounter with a celestial body can significantly improve the efficiency of the swingby maneuver. A parametric analysis of a powered swingby with the moon is carried out to find the optimal combined use of a gravity assist and propulsion. A single impulse, usually applied inside the satellite's sphere of influence, is minimized to achieve an assigned energy with respect to the Earth, for an assigned hyperbolic excess velocity on entering the sphere. Numerical results are presented, but emphasis is placed on the physical reasoning of the suggested strategies that are sometimes different from those found in recent literature. A longer stay on the low-energy, large-deflection hyperbola is always convenient. When geocentric energy is increased, thrust is applied after the passage at the periapsis, with a small misalignment with respect to the spacecraft velocity, to obtain a larger turn angle from the swingby. If reduction of the geocentric energy is required, the selenocentric energy is increased or reduced, depending on the magnitude of the inbound hyperbolic excess velocity, and the corresponding strategies are rather different.

Nomenclature

r_i = position where impulse is applied
$r_{\min}$ = minimum allowable radial distance
r_p = hyperbola periapsis
V_b = secondary-body velocity
V_p = circular velocity at hyperbola periapsis
V_∞ = hyperbolic excess velocity
α = angle between impulse and velocity
ΔE = energy increment in main-body frame
ΔV = velocity impulse
δ = half-turn angle of hyperbola
Θ = total turn angle
Θ_i = velocity turn due to velocity impulse
ϑ = angle between V_∞ and V_b
μ = secondary-body gravitational constant
ν = true anomaly at impulse
Φ = turn angle
φ = flight-path angle at impulse
ψ = inbound hyperbola orientation

Subscripts

1 = inbound hyperbola
2 = outbound hyperbola

Introduction

IN the exploration of the solar system, complex trajectories are often used to increase payload and sometimes guarantee the mission feasibility. Before reaching its final destination, the spacecraft closely approaches planets that are favorably positioned and able to provide gravity assists. The preliminary analysis of interplanetary missions is not a simple task. To accomplish this, we usually apply an indirect method while being aware that the optimization must concern the whole mission. The powered flyby is seldom analyzed in literature; the relevant articles[1–3] have searched for a separate optimization of this single maneuver. In the authors' opinion it is an inconclusive exercise that cannot explain why a crewed Mars mission,[4] i.e., a round-trip fast trajectory, is improved by exploiting an unpowered Venus flyby, whereas the synergetic use of gravitational force and propulsion is instead advisable in other circumstances, for instance, in some missions that use Earth gravity assist.[5]

Nevertheless, this paper, which originates from a recent article,[3] analyzes an isolated flyby maneuver and discusses simple strategies to obtain some improvement by means of engine thrust. In particular, Prado[3] provides a simple and clear description of the powered swingby; a planar maneuver is considered under the patched-conic approximation. The problem is identified by means of three independent variables: magnitude and direction of the inbound hyperbolic excess velocity and periapsis radius of the inbound hyperbola. Prado's paper, in this respect, differs from previous works,[1,2] which assumed the magnitude of the inbound and outbound hyperbolic excess velocities and the angle between them as the independent variables. An impulse is applied at the periapsis to instantaneously change the spacecraft velocity. The magnitude of the impulse and the angle between the impulse and spacecraft velocity are the parameters used to maximize or minimize the specific energy of the spacecraft with respect to the primary body. Little attention is given to the minimum height above the surface of the secondary body. The same analysis is carried out in this paper, but a minimum-height constraint is strictly enforced, whereas no restriction concerns the position where the single velocity impulse is applied.

Statement of the Problem

It is assumed that a celestial body has a velocity V_b on a circular orbit around a primary body. The spacecraft moves in the same plane and uses a close encounter with the secondary body to change its specific energy with respect to the main body. The patched-conic model is used to analyze the swingby; just one impulse, that instantaneously changes the spacecraft velocity, is permitted during the maneuver. The problem (Fig. 1) is identified by three independent parameters: the minimum allowable distance from the secondary body $r_{\min}$, the magnitude $V_{\infty 1}$ of the hyperbolic excess velocity of the spacecraft when approaching the body, and the approach angle ϑ_1, which is assumed to be negative if a positive variation of energy is required from the swingby, and vice versa.

In unpowered flybys, the hyperbolic excess velocity on leaving the sphere of influence of the secondary body has the same magnitude as on entering the sphere ($V_{\infty 2} = V_{\infty 1}$), but its direction $\vartheta_2 = \vartheta_1 + \Theta$ is rotated by an angle $\Theta = 2\delta_1$. The angle

$$\delta = \sin^{-1}\left(\frac{V_p^2}{V_p^2 + V_\infty^2}\right) \tag{1}$$

between the hyperbola asymptote and the horizon at the periapsis only depends on V_∞ and on the periapsis radius r_p, via the corresponding circular velocity $V_p = \sqrt{(\mu/r_p)}$. Note that a large velocity

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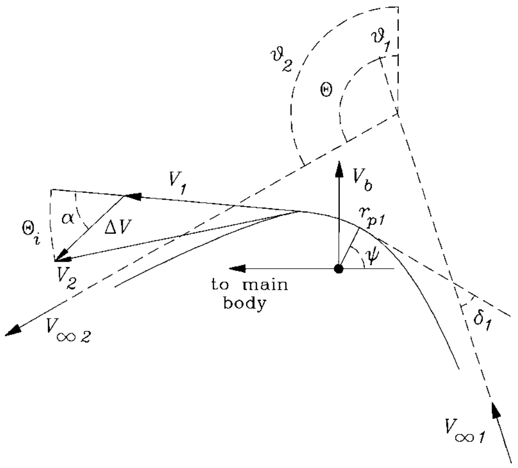

Fig. 1 Geometry of a powered swingby maneuver.

turn is obtained for a low hyperbolic excess velocity and when the periapsis radius is fixed at the minimum allowable value.

Once $V_{\infty 1}$ and r_{p1} are fixed, the variation of the energy with respect to the primary body only depends on the approach angle ϑ_1. By introducing the angle $\psi = \vartheta_1 + \delta_1$ that the hyperbola axis makes with the line that connects the primary to the secondary body, one easily obtains

$$\Delta E = -2 V_b V_{\infty 1} \sin \delta_1 \sin \psi \tag{2}$$

The maximum energy increment is obtained when $\psi = -90$ deg and the maximum reduction when $\psi = 90$ deg.

In powered flybys, engine thrust is used inside the secondary-body sphere of influence with the purpose of magnifying the effects of the nonpropelled flyby, either to increase or to decrease the energy of the spacecraft with respect to the primary body. The energy variation is now

$$\Delta E = \tfrac{1}{2}[V_{\infty 2}(V_{\infty 2} + 2V_b \cos \vartheta_2) - V_{\infty 1}(V_{\infty 1} + 2V_b \cos \vartheta_1)] \tag{3}$$

The rotation $\Theta = \delta_1 + \delta_2 + \nu_1 - \nu_2$ may be considered as the sum of the velocity turns

$$\Phi_1 = \delta_1 + \nu_1 - \varphi_1 \tag{4}$$

$$\Phi_2 = \delta_2 - \nu_2 + \varphi_2 \tag{5}$$

$$\Theta_i = \varphi_1 - \varphi_2 \tag{6}$$

which are, respectively, provided by the movement on either hyperbola and by the application of the velocity impulse: δ is given by Eq. (1), whereas ν and φ are, respectively, the true anomaly and the flight-path angle at the impulse.

Energy Increment

Powered flybys in the Earth–moon system ($V_b = 1.02$ km/s and $\mu = 4900$ km^3/s^2) are considered in Prado's paper.[3] The periapsis of the inbound hyperbola, where the impulse is applied, is fixed ($r_{p1} = 1900$ km) but no constraint on the minimum radius is present. The other data are $V_{\infty 1} = 1$ km/s and $\psi = -90$ deg, i.e., the value corresponding to the optimal unpowered flyby. The energy increment is analyzed by assuming the magnitude of the impulse ΔV and the angle α between the impulse and spacecraft velocity as parameters.

A different presentation of Prado's results is given in Fig. 2 (curves P) for an assigned velocity increment inside the moon's sphere of influence ($\Delta V = 1$ km/s). The most interesting result is that the best direction of the impulse is not parallel to the spacecraft velocity but is almost 20 deg rotated toward the moon to increase the turn angle. The spacecraft is moved on a higher energy hyperbola, whose periapsis $r_{p2} < r_{p1}$ is also shown in Fig. 2. For the problem described in this section, it is always useful to increase the selenocentric energy, i.e., $V_{\infty 2} > V_{\infty 1}$. Energetic considerations would suggest applying a periapsis impulse parallel to the velocity but, due to the increased

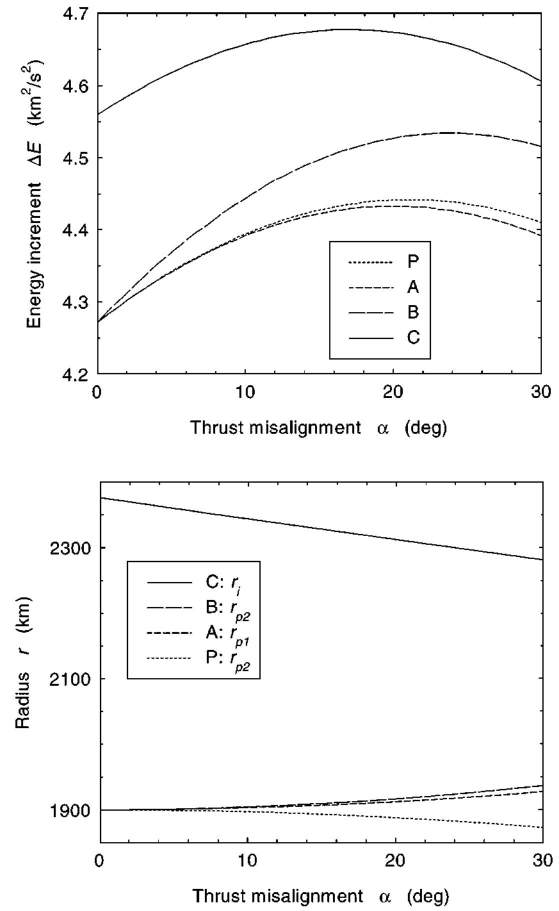

Fig. 2 Prado's results (P) compared to height-constrained maneuvers: impulse at inbound periapsis (A), outbound periapsis (B), and free position (C); $\Delta V = 1$ km/s.

energy, the velocity turn, which is obtained during the motion on the exit hyperbola, would be too low. The velocity turn Θ_i, which is produced by thrust misalignment α, profitably counterbalances the lower $V_{\infty 2}$.

To analyze different strategies, the present paper does not place any restriction on the position where the impulse is applied; comparisons become more significant if the minimum-height constraint $r_{\min} = 1900$ km replaces Prado's similar condition on r_{p1}. For the sake of comparisons, the angle $\vartheta_1 = -\pi/2 - \delta_1$ is computed using Eq. (1) with $r_{p1} = r_{\min}$ and kept constant. The curves A in Fig. 2 describe the powered swingby when the impulse is applied at the periapsis of the inbound hyperbola, but the periapsis of the outbound hyperbola is fixed to the minimum value ($r_{p1} > r_{p2} = r_{\min}$). The energy gain is not appreciably degraded.

However, larger deflections are obtained when the spacecraft flies on lower energy hyperbolas. For maximum geocentric energy, it is better to postpone the passage to the high-energy hyperbola; if the impulse is performed, for the sake of simplicity, at either periapsis, a simple one-parameter analysis shows that better performance (curves B in Fig. 2) is obtained by applying the impulse at the periapsis of the high-energy hyperbola, some degrees after the passage by the periapsis of the low-energy hyperbola. Figure 2 also shows the radius $r_i = r_{p2}$, where, in this case, the impulse is applied.

It is preferable to spend even more time on the low-energy inbound hyperbola (whose periapsis is fixed to the minimum allowable value) and to move to the high-energy outbound hyperbola by applying the thrust in the same direction as the spacecraft velocity. A small misalignment of the thrust (curves C in Fig. 2) is, however, recommended to increase the turn angle without any significant loss, in terms of the selenocentric energy increment. This strategy is the best compromise between the requirements of limiting gravitational and thrust-misalignment losses and obtaining a sufficient velocity turn.

The best strategy is further analyzed in Fig. 3, which shows α and r_i for different magnitudes of the applied impulse. For the lowest

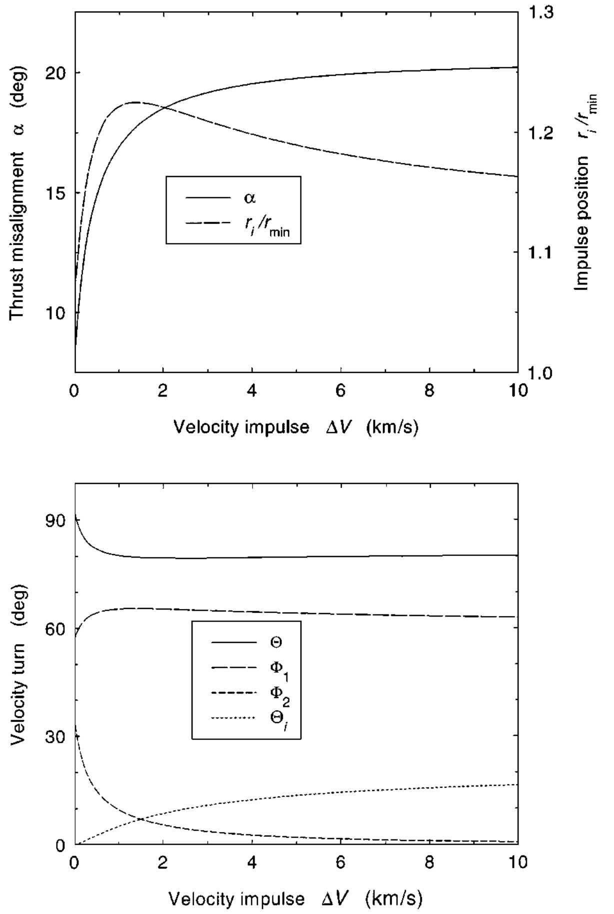

Fig. 3 Analysis of the best strategy for energy increment.

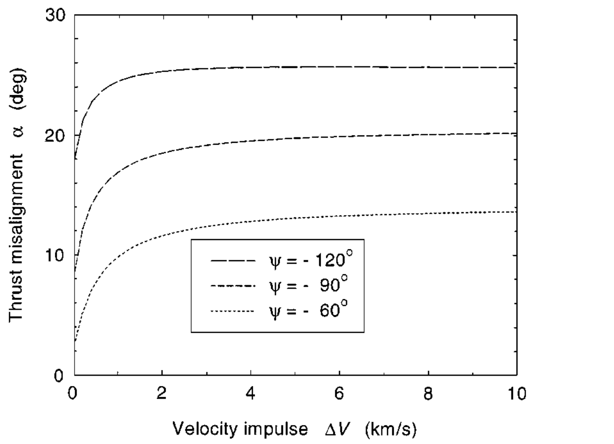

Fig. 4 Effect of the approach angle $\vartheta_1 = \psi - \delta_1$.

impulses, the change of the selenocentric energy is small, and the velocity turn is almost the same as in the unpowered flyby, which presents the optimal rotation, due to the particular value assumed for the independent variable ψ. It is convenient to pay more attention to the selenocentric energy than to the velocity turn; therefore, thrust is applied near the periselenium and almost parallel to the velocity. The energy of the outbound hyperbola rapidly grows with the impulse magnitude, and the corresponding velocity turn Φ_2 is reduced. It is necessary to partially recover the lost rotation by increasing the thrust misalignment α and the stay on the low-energy hyperbola, that is, r_i (with a further benefit for Θ_i). For the highest impulses, when Φ_1 has become a weak function of r_i whereas Θ_i increases with ΔV, even though α is constant, the interest in increasing $V_{\infty 2}$ prevails and r_i again decreases.

Note some results are related to the input data. In particular, the numerical value of the angle α depends on the angle ψ, which defines the orientation of the unpowered hyperbola. If the hyperbola, and the departure asymptote, are rotated toward the moon velocity vector, e.g., $\psi = -60$ deg, a lower Θ is needed and less rotation is obtained by means of thrust misalignment (see Fig. 4).

Energy Reduction

A similar maneuver, but with $\psi = 90$ deg, is also analyzed by Prado.[3] The unpowered swingby produces the effect of reducing energy; the engine thrust is profitably used if this effect is enhanced. Attention is consistently paid, herein, to the maneuvers that use thrust to minimize the spacecraft energy with respect to the primary body. The problem is rather different and more complex than in the preceding section; a lower boundary on the geocentric energy exists in this case and is reached when the outbound hyperbolic excess velocity is equal to the moon's velocity but is in the opposite direction. Figure 5 presents the minimum impulse, which is necessary to attain this boundary, as a function of the inbound hyperbolic excess velocity (the varying angle $\vartheta_1 = \pi/2 - \delta_1$ is computed by assuming $r_{p1} = r_{\min}$). For the lowest and highest $V_{\infty 1}$, the main task is to achieve the required exit velocity $V_{\infty 2} = V_b$, by increasing the hyperbola energy for a low $V_{\infty 1}$ and decreasing it for a high $V_{\infty 1}$. The impulse is applied inside the sphere of influence (solid lines) to take energetic benefit from the proximity to the secondary body. On the contrary, when $V_{\infty 1}$ is close to V_b, the task of the impulse is that of rotating the hyperbolic excess velocity, and thrust is better used outside the sphere of influence (dotted lines), where the spacecraft velocity is lowest. A larger velocity turn is always obtained if the spacecraft passes by the periselenium on the low-energy hyperbola. It is convenient to apply the impulse after the closest approach, if $V_{\infty 1} < V_b$, and before it, in the reverse case.

It is opportune to underline another point that is not apparent in Prado's paper.[3] During a swingby that is aimed at reducing spacecraft energy, it is not convenient to apply a larger velocity impulse than the limit shown in Fig. 5. When $V_{\infty 1}$ is close to V_b, the limit is actually lower. Figure 6 presents the energy reduction for different

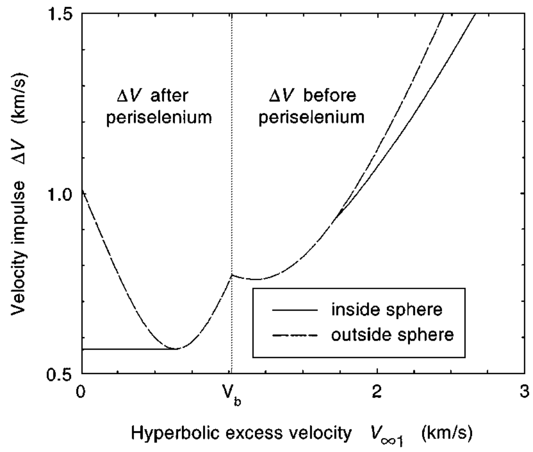

Fig. 5 Swingby for maximum energy reduction.

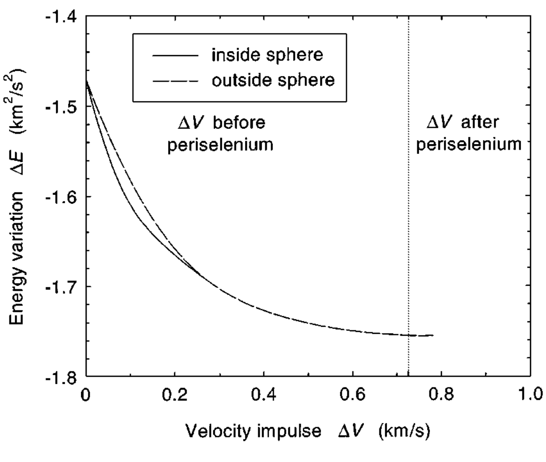

Fig. 6 Energy reduction; $V_{\infty 1}$ = 1 km/s.

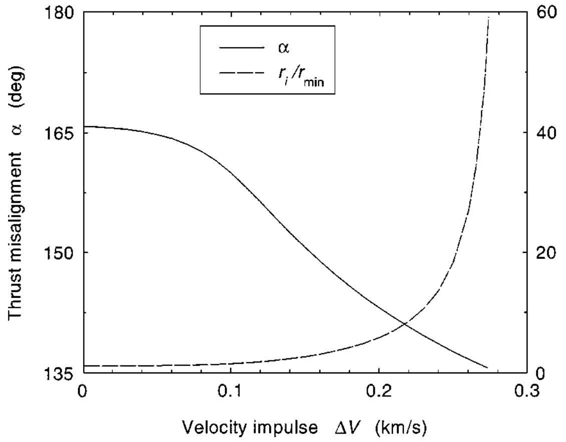

Fig. 7 Suggested strategy for energy reduction; $V_{\infty 1}$ = 1 km/s.

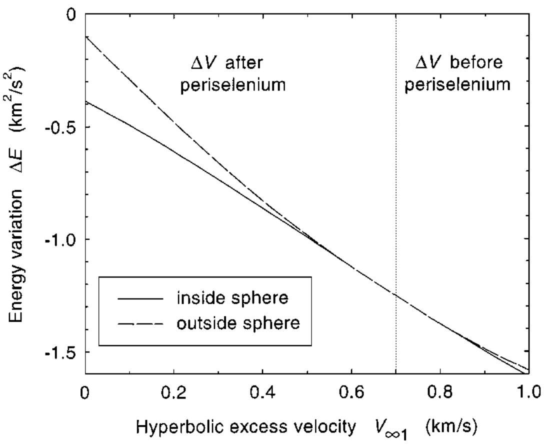

Fig. 8 Energy reduction; Δ V = 0.1 km/s.

magnitudes of the applied impulse ($V_{\infty 1} = 1$ km/s). Before achieving the maximum energy reduction, the impulse has moved outside the sphere of influence before the swingby, and this outcome would suggest to search for a better approach trajectory under the sole influence of the main body. In Fig. 7, attention is given only to the powered manuever inside the sphere of influence. Engines are fired before the periselenium to reduce velocity ($\alpha > 90$ deg) and favor the rotation of the hyperbolic excess velocity. When a larger ΔV is available, it is used to increase the turn angle by misaligning thrust and velocity instead of further reducing $V_{\infty 2}$.

Figure 8 is analogous to Fig. 5, but the energy reduction that is attainable by $\Delta V = 0.1$ km/s is analyzed. Previous conclusions are confirmed, but the range of application of the powered flyby appears to be larger in this more realistic case.

The arrival hyperbolic excess velocity should be rather low in a planetary flyby for an actual mission to interior planets. A powered flyby with $V_{\infty 1} = 0.1$ km/s is considered in Fig. 9. This case closely resembles the powered swingby used to increase energy. In both cases, the spacecraft energy in the secondary-body frame must be increased, and thrust is, therefore, used after the periapsis of the

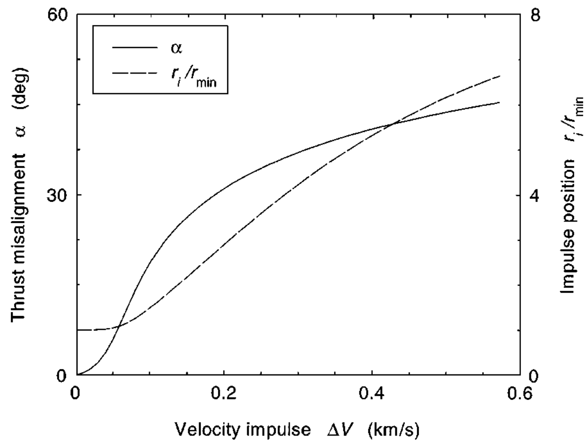

Fig. 9 Suggested strategy for energy reduction; $V_{\infty 1}$ = 0.1 km/s.

inbound hyperbola. Larger velocity impulses are applied with larger misalignments and at greater distances from the secondary body until the limit value for the geocentric energy is reached.

In this section, the specific energy in the primary-body reference frame is reduced (the limit value corresponds to no kinetic energy). The spacecraft performs the swingby and moves toward the main body while gaining velocity; a better opportunity for using thrust to reduce energy is highly probable. This is further support for the opinion that a powered flyby should not be optimized independently of the remaining trajectory.

Conclusions

A parametric analysis of a single-impulse powered swingby has been carried out, and suggestions concerning the most convenient position and direction of the velocity impulse have been provided. The suggested strategy improves the good performance of a maneuver that has been recently proposed in literature. The results depend on the numerical assumptions concerning the magnitude and direction of the inbound hyperbolic excess velocity, which are independent variables in the present position of the problem but are subject to optimization in practical mission analyses. Nevertheless, this study is considered to be preparatory but indispensable by the authors, who aim to introduce powered swingby into their simple indirect procedure for the preliminary analysis of interplanetary missions.

References

[1]Gobetz, F. W., "Optimum Transfers Between Hyperbolic Asymptotes," *AIAA Journal*, Vol. 1, No. 9, 1963, pp. 2034–2041.

[2]Walton, J. M., Marchal, C., and Culp, R. D., "Synthesis of the Types of Optimum Transfers Between Hyperbolic Asymptotes," *AIAA Journal*, Vol. 13, No. 8, 1975, pp. 980–988.

[3]Prado, A. F. B. A., "Powered Swingby," *Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 5, 1996, pp. 1142–1147.

[4]Casalino, L., Colasurdo, G., and Pastrone, D., "Optimization Procedure for Preliminary Design of Opposition-Class Mars Missions," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 1, 1998, pp. 134–140.

[5]Casalino, L., Colasurdo, G., and Pastrone, D., "Optimization of Δ V-Earth-Gravity-Assist Trajectories," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 6, 1998, pp. 991–995.